

6.4

Logarithmic Functions



DEFINITION

The **logarithmic function to the base a** , where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as “ y is the logarithm to the base a of x ”) and is defined by

$$y = \log_a(x) \text{ if and only if } x = a^y$$

The domain of the logarithmic function

$$y = \log_a x \text{ is } x > 0.$$

Convert Exponential to Logarithm

<u>exp.</u>	→	<u>log.</u>
$5^2 = 25$		$\log_5 25 = 2$
$6^3 = 216$		$\log_6 216 = 3$
$2^{-3} = \frac{1}{8}$		$\log_2 \left(\frac{1}{8}\right) = -3$
$537^0 = 1$		$\log_{537} 1 = 0$

2 Special Logarithms

Common Logarithm (base 10)

log button on calculator

$\log_{10}x$ is always written as log x

(if no base is written, it is understood to be base 10)

Natural Logarithm (base e)

ln button on calculator

$\log_e x$ is always written as ln x

Example 1: Changing Exponential Statements to Logarithmic Statements

Change each exponential expression to an equivalent expression involving a logarithm.

(a) $1.2^3 = m$

(b) $e^b = 9$

(c) $a^4 = 24$

$$\log_{1.2} m = 3$$

$$\ln 9 = b$$

$$\log_a 24 = 4$$

Example 2: Changing Logarithmic Statements to Exponential Statements

Change each logarithmic expression to an equivalent expression using exponents.

a.) $\log_a 4 = 5$

$$a^5 = 4$$

b.) $\ln b = -3$

$$e^{-3} = b$$

c.) $\log_3 7 = c$

$$3^c = 7$$

d.) $\log 7 = d$

$$10^d = 7$$

Example 3: Evaluate Logarithmic Expressions

Evaluate each logarithmic expression.

$$\begin{array}{ll} \text{(a) } \log_3 81 = 4 & \text{(b) } \log_2 \frac{1}{8} = -3 \\ \downarrow & \downarrow \\ 3^? = 81 & 2^? = \frac{1}{8} \\ 3^4 = 81 \checkmark & 2^3 = 8 \\ & \text{So } 2^{-3} = \frac{1}{8} \checkmark \end{array}$$

Your Turn

(a) $\log_2 32 = 5$

$$2^5 = 32 \checkmark$$

(b) $\log_{10} 1000$

$$\log 1000 = 3$$

$$10^3 = 1000 \checkmark$$

(c) $\log_4 \frac{1}{16} = -2$

$$4^{-2} = \frac{1}{4^2} \checkmark$$

Ex 4: Use your calculator to evaluate.
Round to 3 places as needed.

a.) $\frac{\log 35}{10} \approx 0.154$

b.) $3\ln 7 \approx 5.838$

c.) $\frac{(\ln 7 + \ln 2)}{(\log 8 - \log 4)} \approx 8.767$

Determining the Domain of a Logarithmic Function

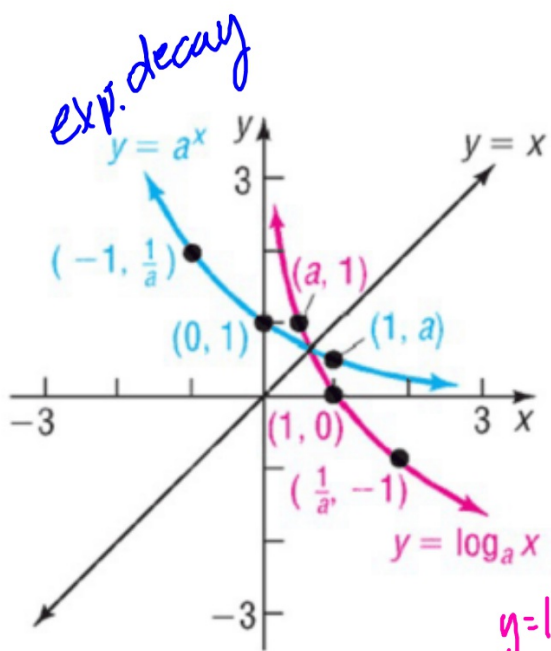
Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$

Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$

$$y = \log_a x \quad (\text{defining equation: } x = a^y)$$

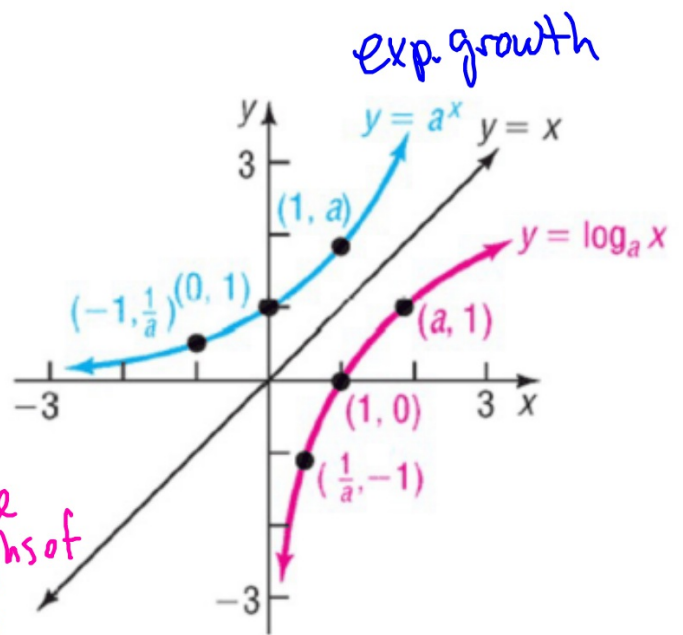
$$\text{Domain: } 0 < x < \infty \quad \text{Range: } -\infty < y < \infty$$

\rightarrow $(0, \infty)$
 \rightarrow $(-\infty, \infty)$
* input into log. must be positive



(a) $0 < a < 1$

the graphs of $y = \log_a x$ have v.A. @ $x=0$ and x -int: $(1, 0)$



(b) $a > 1$

Example 5: Finding the Domain of a Logarithmic Function

Find the domain of each logarithmic function.

*argument of the log.
must be > 0*

(a) $f(x) = \log_3(x-2)$

$x-2 > 0$

$D: x > 2$

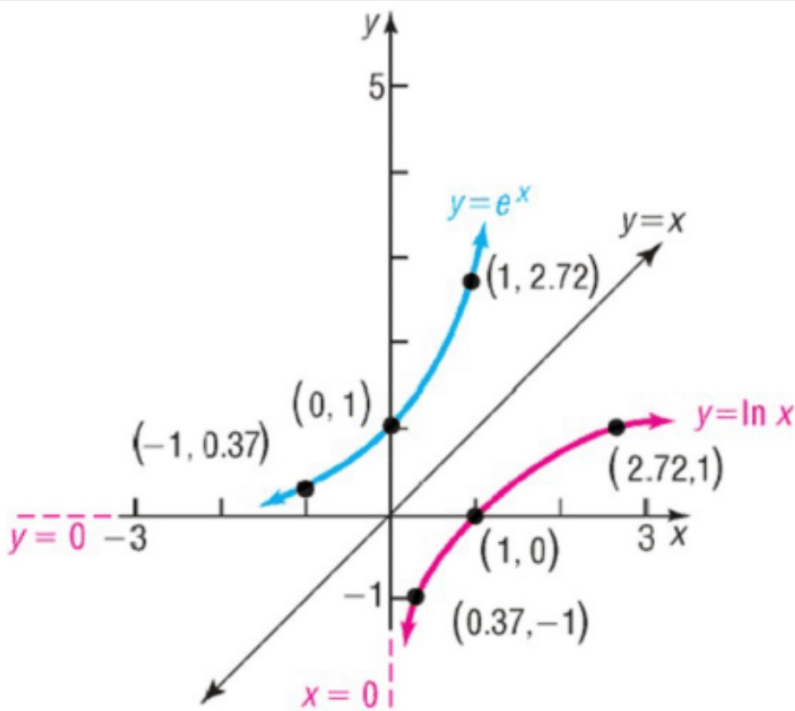
$D: (2, \infty)$

(b) $F(x) = 9 - 8\log_2\left(\frac{x}{8} - 4\right)$

$\frac{x}{8} - 4 > 0$

$\frac{x}{8} > 4$

$D: x > 32$



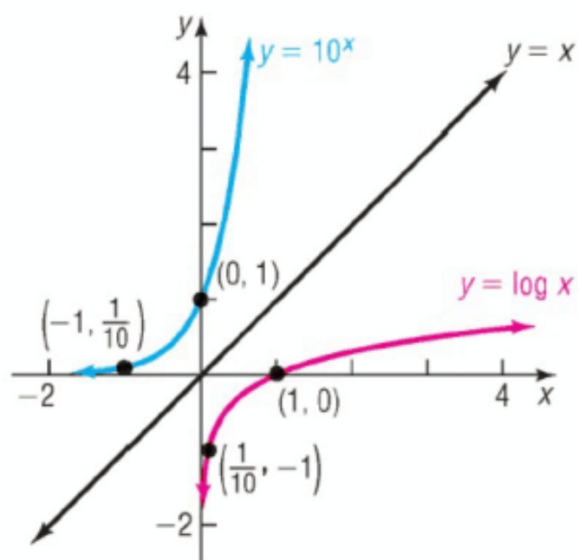
x	$\ln x$
$\frac{1}{2}$	-0.69
2	0.69
3	1.10

Natural Logarithm Function

$$y = \ln x \quad \text{if and only if} \quad x = e^y$$

Common Logarithm Function

$$y = \log x \quad \text{if and only if} \quad x = 10^y$$



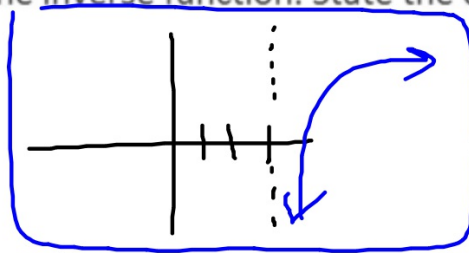
Example 6: Graphing Logarithmic Functions

Graph the functions. Find the inverse function. State the domain and range of each.

(a) $y = \log_2(x - 3)$

$x - 3 > 0$
 $x > 3$

D: $x > 3$
R: all reals

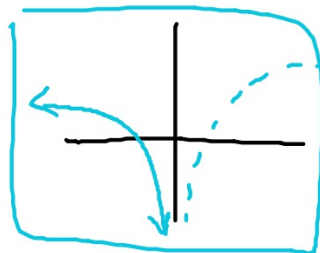


(b) $y = \ln(-x)$

$-x > 0$

D: $x < 0$

R: all reals



(a) continued...

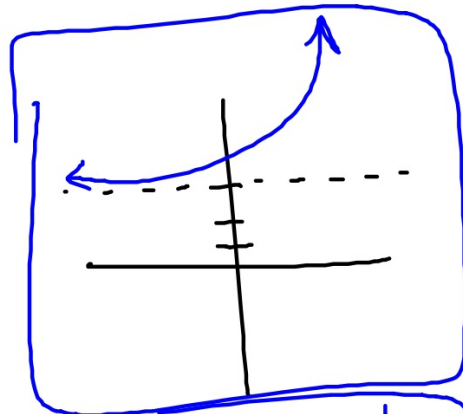
Convert
to
exp.
form

$$x = \log_2(y-3)$$

$$2^x = y-3$$

$$2^x + 3 = y$$

$$f^{-1}(x) = 2^x + 3$$



D: all reals
R: $y > 3$

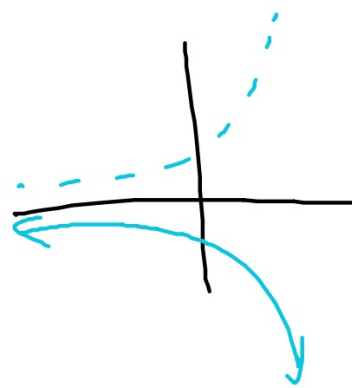
(b) continued...

Convert! $x = \ln(-y)$

$$e^x = -y$$

$$-e^x = y$$

$$f^{-1}(x) = -e^x$$



D: all reals
R: $y < 0$

Example 7:

Solving Logarithmic Equations

Solve:

(a) $\log_3(4x - 7) = 2$

Convert
to
exp.
form

$$3^2 = 4x - 7$$

$$9 = 4x - 7$$

$$16 = 4x$$

$$x = 4$$

(b) $\log_x 64 = 2$

$$x^2 = 64$$

$$x = 8$$

* not ± 8 because x is the base, and base "a" must be positive for log's and exp's